A Direct Spectral Domain Method for Near-Ground Microwave Radiation by a Vertical Dipole Above Earth in the Presence of Atmospheric Refractivity

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Outline

- Background and Motivation
- Environmental and Mathematical Models
- The Spectral Domain Green's Functions
- Sommerfeld Integrals
  - Pole Extraction
  - Hybrid Asymptotic / Numerical Evaluation
- Numerical Experiments
- Future Research

Motivation: Real World Applications

Mesh topology low power networks

- Wireless sensor networks
- Unattended ground sensor networks

Applications

- Intrusion detection
- Localization
- Tomography
- Agricultural Sensing
- Cooperative Cognitive Radio

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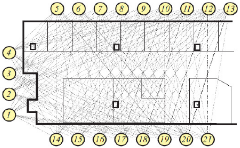

Refractivity

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In non-magnetic media, $n = \sqrt{\varepsilon_r}$

In air, n is a weak function of pressure, temperature, and air composition (H_2O is dominant term, followed by CO_2) as

$$N = (n-1) \times 10^6 = K_1 \frac{P}{T} + K_2 \frac{e}{T} + K_3 \frac{e}{T^2}$$

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If P, T, e are functions of height z above the earth, then so is N.

Refractive effects are known to be significant (dominant propagation mechanism in some cases) in classical radar scenarios, i.e. tens of kilometers of range and kilometers of height over the ocean.

Near-Ground Temperature and Humidity

Dynamics: boundary-layer fluid mechanics + solar forcing.

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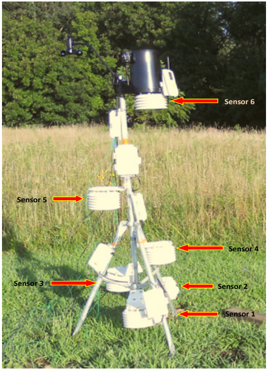
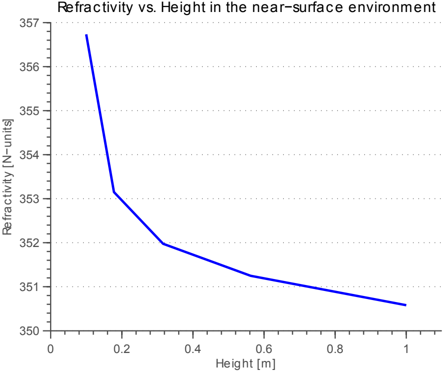
We use some measurements

$$(P(z), T(z), e(z)) \quad \Rightarrow \quad N = K_1 \frac{P}{T} + K_2 \frac{e}{T} + K_3 \frac{e}{T^2}$$





Maxwell's Equations in Plane-Stratified Media

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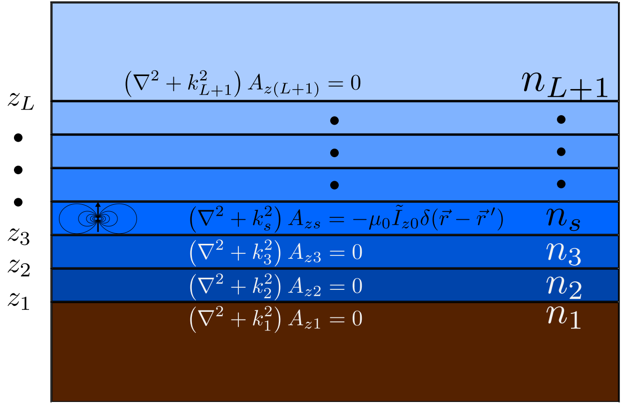
Time-harmonic Maxwell's equations ($\exp(-j\omega t)$), Lorenz gauge potentials, in multilayered media:

$$\left(\nabla^2 + k_\ell^2\right) \mathbf{A} = -\mu_0 \mathbf{J}$$

Tangential $\mathbf{E}, \mathbf{H}$ continuous at z_ℓ

Sommerfeld radiation condition

$$\ell = \{1, 2, \ldots L+1\}$$



The Spectral Domain Green's Function

$$\left(\nabla^2 + k_\ell^2\right) A_{z\ell} = -\mu_0 \tilde{I}_{z0} \delta(z - z')$$

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Reduces the PDE to an ODE

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With solution

$$\tilde{A}_{z\ell} = j \frac{\mu_0 \tilde{I}_{z0}}{8\pi^2} \times \begin{cases} \frac{e^{jk_{z\ell}|z - z'|}}{k_{z\ell}} + R_\ell^+ e^{jk_{z\ell}(z - z_{\ell-1})} + R_\ell^- e^{-jk_{z\ell}(z - z_\ell)}, & \text{source layer} \\ R_\ell^+ e^{jk_{z\ell}(z - z_{\ell-1})} + R_\ell^- e^{-jk_{z\ell}(z - z_\ell)}, & \text{other layers} \end{cases}$$

Spectral Domain Boundary Conditions

Tangential continuity of $\mathbf{E}, \mathbf{H}$ + Sommerfeld radiation condition:

$$\left(\tilde{A}_{z\ell} = \tilde{A}_{z(\ell+1)}\right)_{z=z_\ell} \qquad \left(\frac{1}{n_\ell^2}\frac{\partial \tilde{A}_{z\ell}}{\partial z} = \frac{1}{n_{\ell+1}^2}\frac{\partial \tilde{A}_{z(\ell+1)}}{\partial z}\right)_{z=z_\ell}$$

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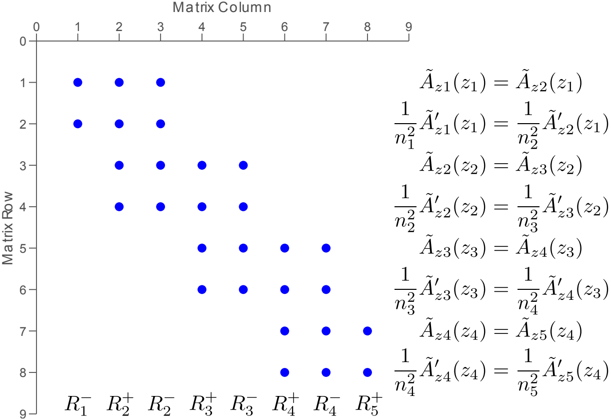

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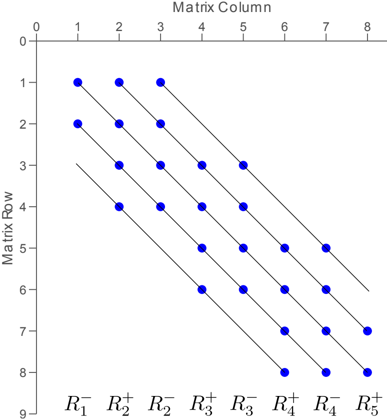

$$\tilde{A}_{z1}(z_1) = \tilde{A}_{z2}(z_1)$$

$$\frac{1}{n_1^2}\tilde{A}'_{z1}(z_1) = \frac{1}{n_2^2}\tilde{A}'_{z2}(z_1)$$

$$\tilde{A}_{z2}(z_2) = \tilde{A}_{z3}(z_2)$$

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Sommerfeld Integral and Quadrature

Fourier integral simplifies to Sommerfeld integral:

$$\iint_{(k_x,k_y)} \tilde{A}_z(k_x,k_y)\exp\left(j(k_x x + k_y y)\right) dk_x dk_y \Rightarrow \int_0^\infty \tilde{A}_z(k_\rho) J_0(k_\rho \rho) k_\rho dk_\rho$$

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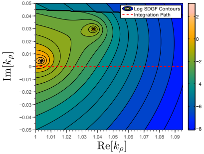
Singularities in complex-k_ρ plane



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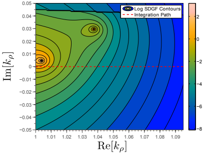
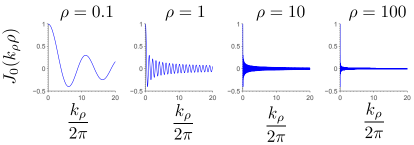


Oscillatory nature of $J_0(k_\rho \rho)$



Singularity Extraction and Modal Solutions

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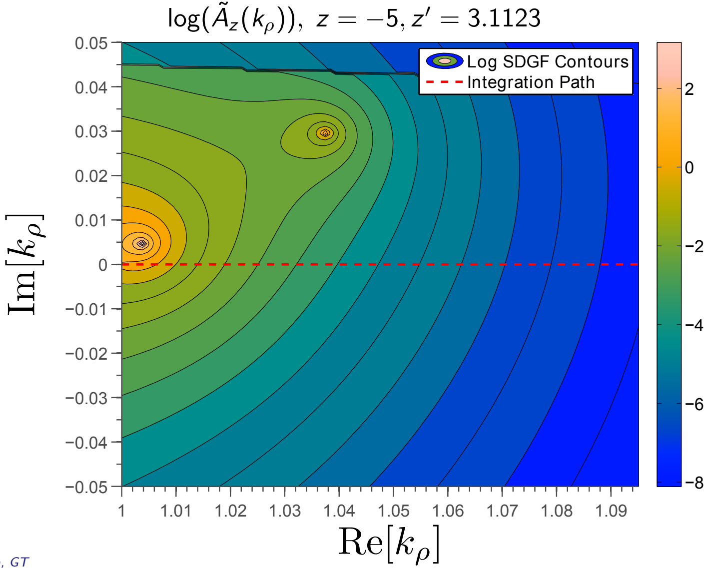

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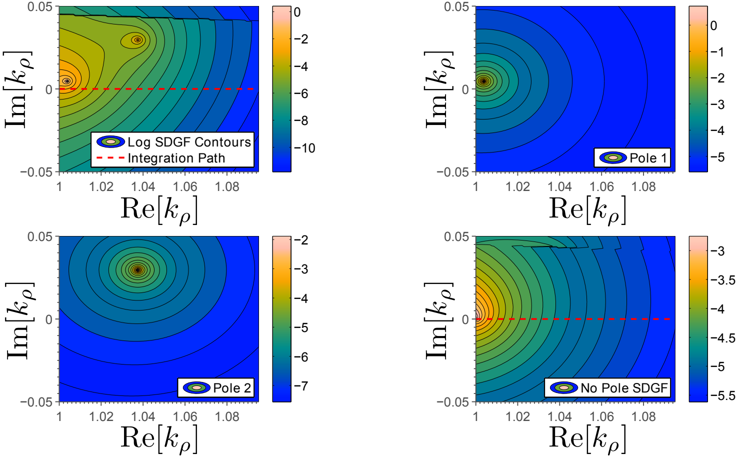

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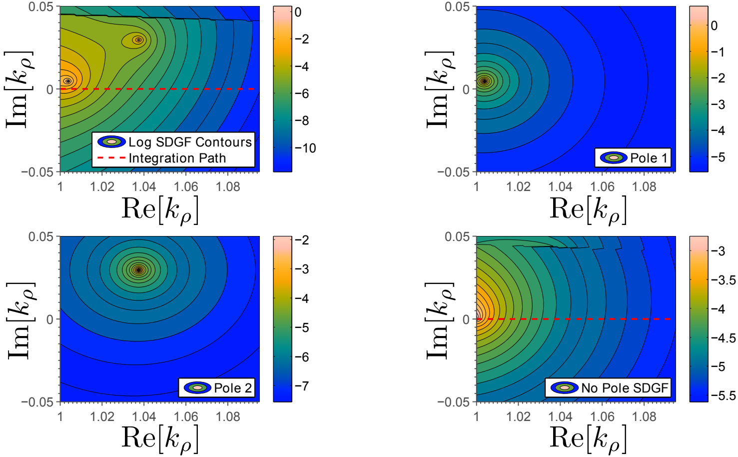
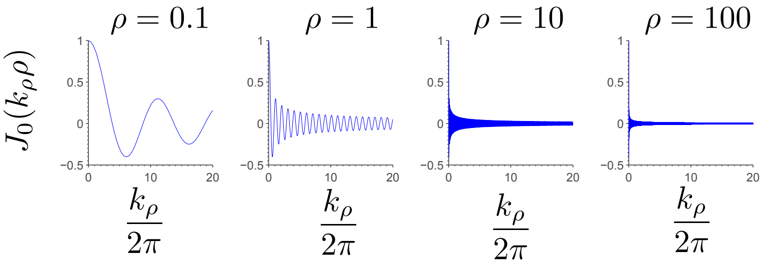
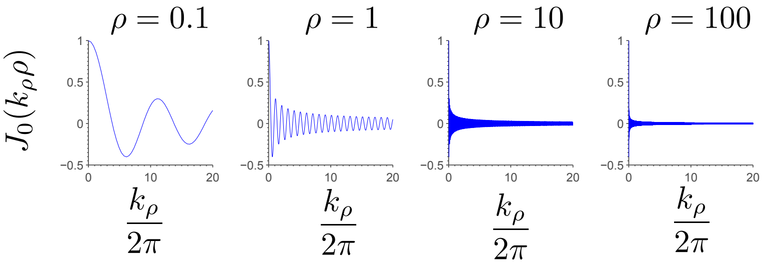


$$\int_0^\infty \frac{f(z)}{k_\rho^2 - k_p^2} J_0(k_\rho \rho)\, k_\rho \, dk_\rho = \frac{j\pi}{2} f(z) H_0^{(1)}(k_p \rho)$$

Handling Oscillatory Integrals

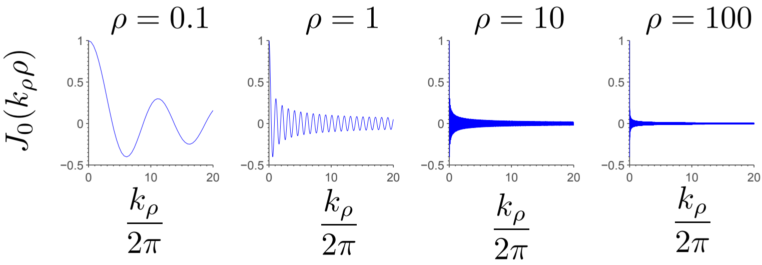




Usual Approaches

Contour deformation to less oscillatory paths

Handling Oscillatory Integrals



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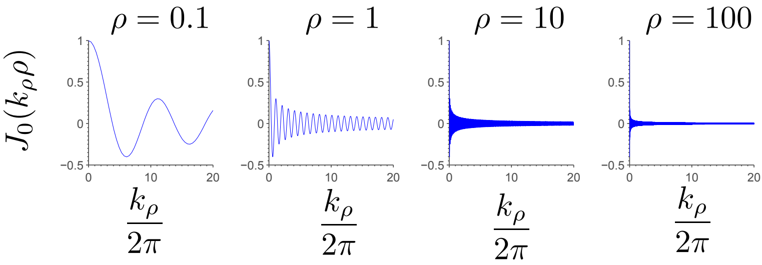

Usual Approaches
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New approach: account for the oscillation in the quadrature rule weights.
Takes numerical samples and returns asymptotic ($\rho \to \infty$) results:
Filon-Clenshaw-Curtis Quadrature.

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$$\tilde{A}_z(k_\rho) \approx \sum_{m=0}^{M} a_m T_m(k_\rho)$$

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There is a recurrence relationship among T_m polynomials (and thus among the weights). Solving the recurrence numerically for any M and ρ is its own problem, solved by V. Domínguez in the FCC package.

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Numerically stable implementation for $\int \tilde{A}_z(k_\rho) \exp(jk_\rho \rho) dk_\rho$, but $J_0(k_\rho \rho) \approx \sqrt{\frac{1}{2\pi k_\rho \rho}} \left(e^{jk_\rho \rho - j\frac{\pi}{4}} + e^{-jk_\rho \rho + j\frac{\pi}{4}} \right)$ for large ρ

Putting it All Together: Numerical Results 1

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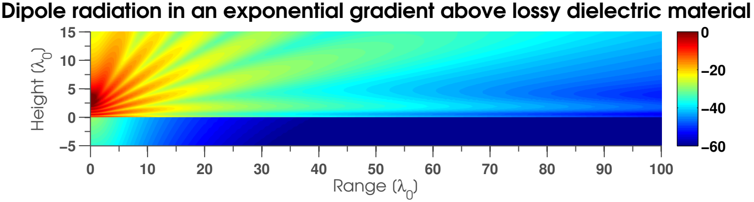
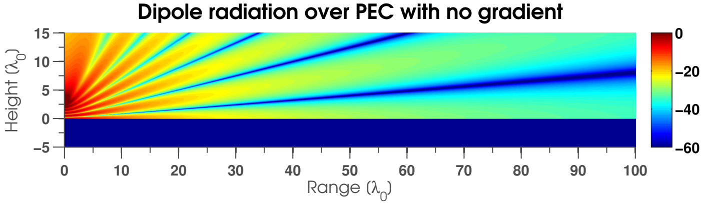
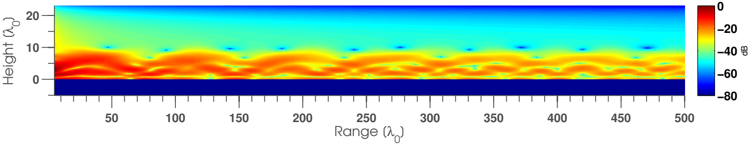


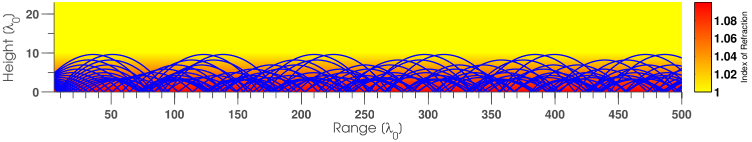


Putting it All Together: Numerical Results 2

Dipole guided modes at 5.8 GHz in a linear gradient above copper



Ray−tracing of guided modes in a linear gradient above copper



Future Research

- Closed form expressions for the quadrature weights
- Horizontal Dipoles and Magnetic Duals
- Scattering formulation based on surface integral equations for terrain
- Validation against measurements

Acknowledgements

- Dr. V. Domínguez of Universidad Pública de Navarra
- This work is sponsored by the US Office of Naval Research

Extra Material

Environmental Model: Plane-Stratified Lossy Dielectric